PNS School of Engineering & Technology

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Internal Assessment Examination-2024 (1st Semester)

Subject : Th-3- Engg. Mathematics-I

Branch : Civil, Mechanical, Electrical, Comp.Sc & ETC Engineering

Time : $1\frac{1}{2}$ Hours F.M. : 20

1. *Answer all the following questions.* [2 x 5]

   (a) *Find the value of* $\sin 480^0$.

   (b) *Draw the graph of* $\sin x$, $-2\pi \leqslant x \leqslant 2\pi$

   (c) *Prove that* $\tan 57^0 = \dfrac{\cos 12^0 + \sin 12^0}{\cos 12^0 - \sin 12^0}$

   (d) *Evaluate :* $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$

   (e) *Evaluate :* $\lim\limits_{x \to \infty} \dfrac{3x^2 + 5x + 7}{4x^2 + 8x - 9}$

2. *Answer all the following questions.* [5 x 2]

   (a) *Show that* $\cos 20^0 . \cos 40^0 . \cos 60^0 . \cos 80^0 = \dfrac{1}{16}$

   (b) *Evaluate :* $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{x}$

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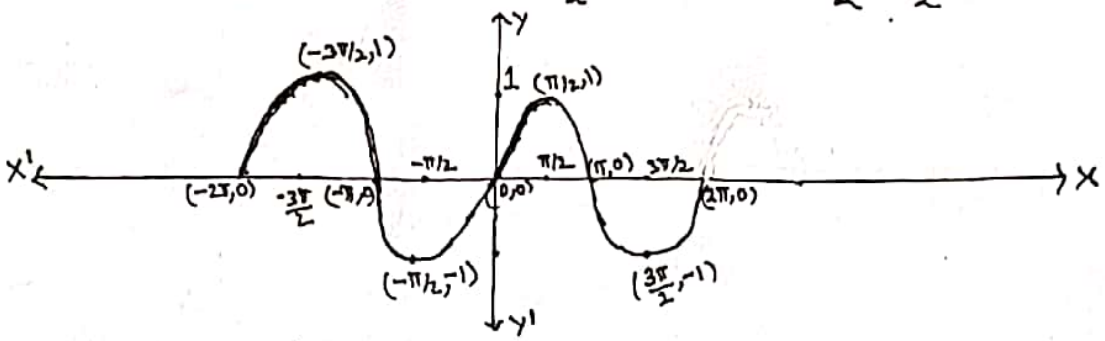
SOLUTION

1(a) $\sin 480^\circ = \sin\left(\frac{5\pi}{2}+30\right) = \cos 30^\circ = \frac{\sqrt{3}}{2}$ $\because 480^\circ \in Q_2$ & $\frac{n\pi}{2}\pm\theta$ form.

(b)



(c) $RHS = \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ} = \frac{1+\tan 12^\circ}{1-\tan 12^\circ}$ Dividing N^r & D^r by $\cos 12^\circ$

$= \frac{\tan 45 + \tan 12}{1-\tan 45 \cdot \tan 12} = \tan(45+12) = \tan 57^\circ = LHS$ □

d. $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{ax}\times ax}{\frac{\sin bx}{bx}\times bx} = \frac{a}{b}\lim_{x\to 0}\frac{\frac{\sin ax}{ax}}{\frac{\sin bx}{bx}}$

$= \frac{a}{b}\cdot\frac{\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx\to 0}\left(\frac{\sin bx}{bx}\right)}$ (As $x\to 0, ax, bx\to 0$) $= \frac{a}{b}\times\frac{1}{1} = \frac{a}{b}$ (Ans)

e. $\lim_{x\to\infty}\frac{3x^2+5x+7}{4x^2+8x-9} = \lim_{x\to\infty}\frac{3+\frac{5}{x}+\frac{7}{x^2}}{4+\frac{8}{x}-\frac{9}{x^2}}$ Dividing N^r & D^r by x^2

$= \frac{3+0+0}{4+0-0} = \frac{3}{4}$ (Ans)

No2 (a) $LHS = \cos 20\cdot\cos 40\cdot\cos 60\cdot\cos 80 = \frac{1}{2}\cos 20\cdot\cos 40\cdot\cos 80$

$= \frac{1}{2}\cos 20\cdot\cos(60-20)\cdot\cos(60+20) = \frac{1}{2}\cos 20\cdot(\cos^2 20 - \sin^2 60)$

$= \frac{1}{2}\cos 20^\circ\left(\cos^2 20 - \left(\frac{\sqrt{3}}{2}\right)^2\right) = \frac{1}{2}\cos 20^\circ\left(\cos^2 20 - \frac{3}{4}\right)$

$= \frac{1}{2}\cos 20^\circ\left(\frac{4\cos^2 20 - 3}{4}\right) = \frac{1}{8}(4\cos^3 20 - 3\cos 20) = \frac{1}{8}\cos(3\times 20)$

$= \frac{1}{8}\cos 60 = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16} = RHS$ □.

(b) $\lim_{x\to 0}\frac{a^x-b^x}{x} = \lim_{x\to 0}\frac{a^x-1+1-b^x}{x} = \lim_{x\to 0}\frac{(a^x-1)-(b^x-1)}{x}$

$= \lim\left\{\frac{a^x-1}{x} - \frac{b^x-1}{x}\right\} = \lim_{x\to 0}\frac{a^x-1}{x} - \lim_{x\to 0}\frac{b^x-1}{x}$

$= \ln a - \ln b = \ln\left(\frac{a}{b}\right)$ (Ans).