UNIT 2

FORCE & MOTION

SCALAR QUANTITY

The quantity which requires only magnitude to define is called scalar quantity.

Ex-Mass , length, time, volume, area, density, energy, temperature, electric charge etc.

VECTOR QUANTITY

The quantity which require both magnitude & direction to define is called vector quantity. Ex- displacement, velocity, acceleration, momentum, Force, Magnetic moment, electric intensity etc.

REPRESENTATION OF VECTOR

A vector quantity is represented by a straight line with an arrow head over it.

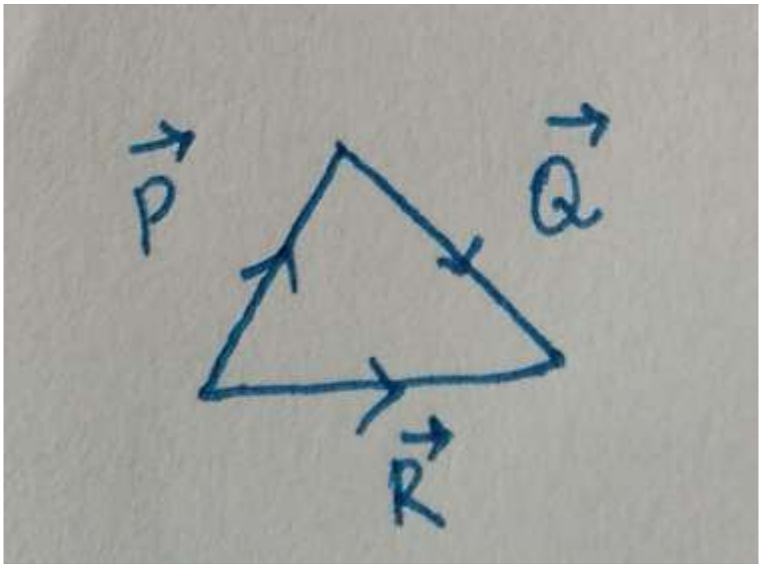
ORTHOGONAL UNIT VECTORS

If Ax, Ay & Az are 3 rectangular components of $\vec{A}$ along x-axis, y-axis and z-axis respectively then

Ax= $\hat{i}$ Ax , Ay = $\hat{j}$ Ay , Az = $\hat{k}$ Az where $\hat{i}$ $\hat{j}$ & $\hat{k}$ are called orthogonal unit vectors or base vector.

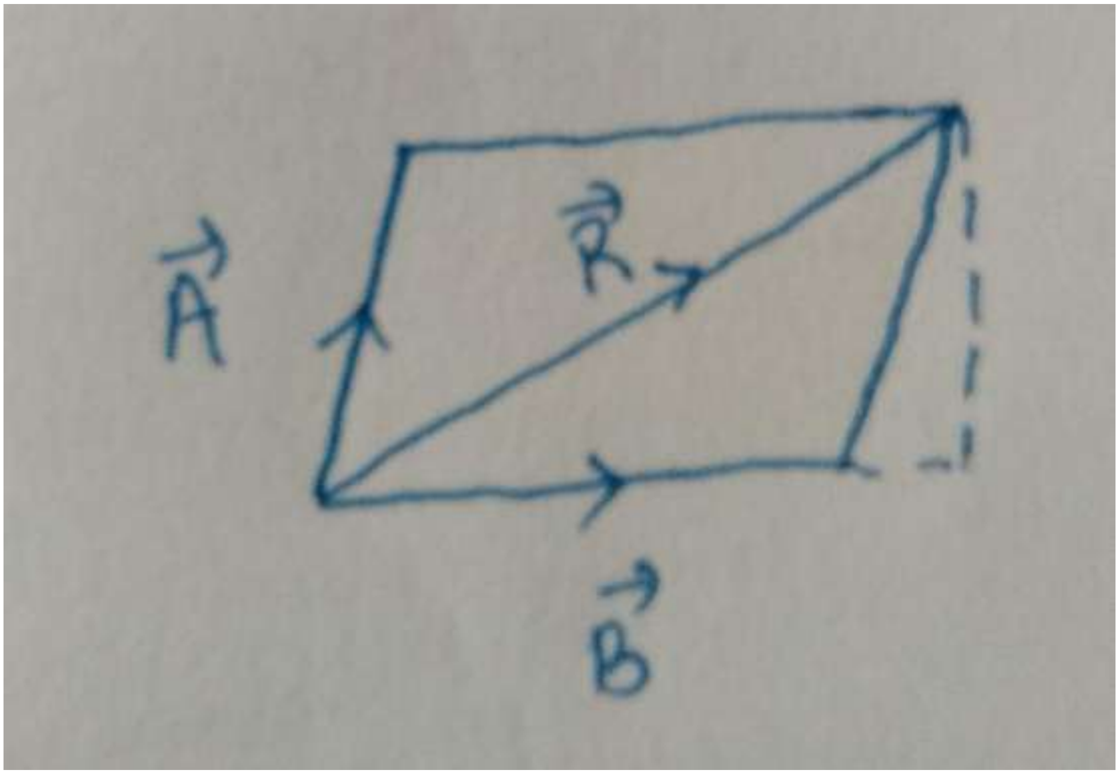
TRIANGLE LAW OF VECTOR ADDITION

It states that if two vectors acting at a point simultaneously be represented in magnitude and direction by the two sides of a triangle taken in order then their resultant vector is represented by the third side of the triangle taken in opposite order.



PARALLELOGRAM LAW OF VECTOR ADDITION

It states that if two vectors acting at a point simultaneously be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant vector is represented by the concurrent diagonal of the parallelogram.



Mathematically

$R^2 = A^2 + B^2 + 2AB \cos \theta$

DOT PRODUCT OF VECTORS / SCALAR PRODUCT

Dot Product of two vectors is defined as Product of their magnitudes and the cosine of the smaller angle between them.

$\vec{A} . \vec{B}$ = AB COS θ

The result of Dot Product of two vectors is scalar so it is also called as scalar product.

Characteristics

i) Distributive: It obey distributive law

ii) commutative: It obey commutative law.

iii). For two perpendicular vectors dot Product is zero If θ =90° then A·B = 0

So $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{i} \cdot \hat{k} = 0$

(iv) If two vector are parallel then dot product is maximum & if two vector are antiparallel then dot product is Minimum

$\vec{A} \cdot \vec{B}$ = AB COS 0° = AB = Maximum (Fore Parallel)

$\vec{A} \cdot \overrightarrow{B}$ = AB COS 180° = -AB = Minimum (Fort Antiparallel)

v) Dot Product of two equal vectons is equal to square of the magnitude of the either vector

$\overrightarrow{A} \cdot \vec{A}$ = AA COS 0° = A^2

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 0$

vi) $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ & $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_Z \hat{k}$ then $\overrightarrow{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

CROSS PRODUCT OF VECTORS/VECTOR PRODUCT

Cross Product of two vectors A & B is defined as another vector C whose magnitude is equal to Product of their individual magnitudes and the sine of the smaller angle between them. It is directed along the normal to the plane containing A & B.

Mathematically $\vec{A} \times \vec{B}$ = AB Sin θ $\hat{n}$ where $\hat{n}$ is the unit vector in a direction perpendicular to Plane of $\overrightarrow{A}$ & $\vec{B}$ It is also called vector product.

Characteristics

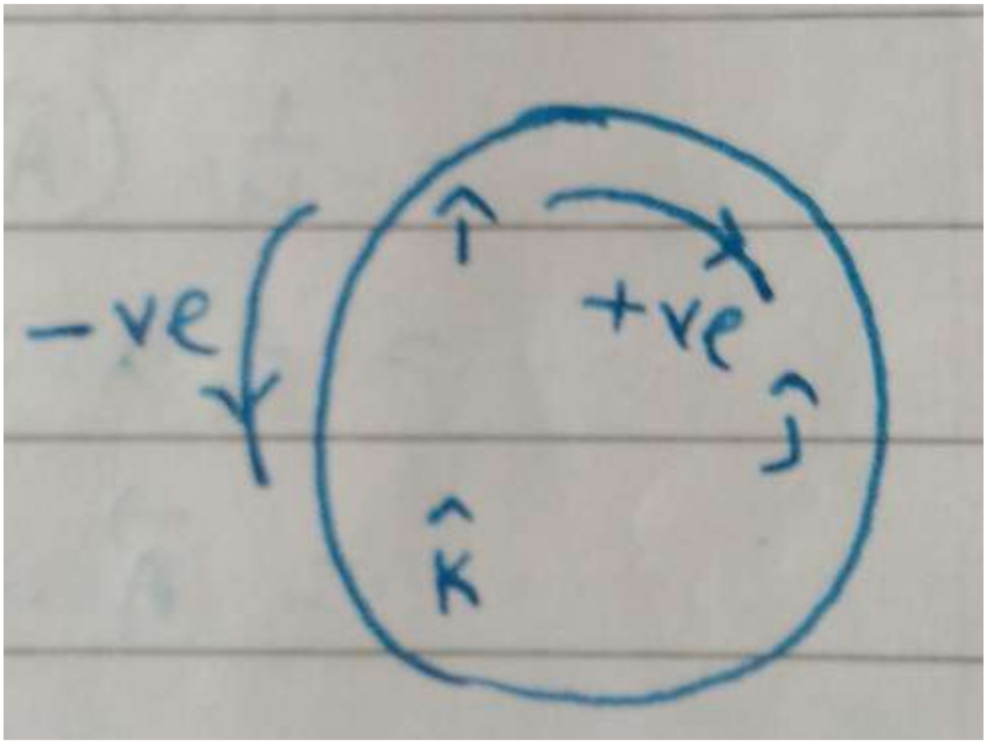
i) Distributive: It obey distributive law.

$\vec{A}$ x ($\vec{B} + \vec{C}$ +........) = $\vec{A}$ X $\vec{B}$ +$\vec{A}$ x $\vec{C}$ + ...

ii) Anti commutative $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ but $\vec{A} \times$ B = - ($\vec{B} \times \vec{A}$)

iii) For two perpendicular vectors cross product is maximum $\vec{A}$ X $\vec{B}$ = AB Sin90° $\hat{n}$ = AB $\hat{n}$

$\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\hat{\jmath} \times \hat{\imath} = -\hat{k}$



$\hat{\jmath} \times \hat{k} = \hat{\imath}$ $\hat{k} \times \hat{\jmath} = -\hat{\imath}$

$\hat{k} \times \hat{\imath} = \hat{\jmath}$ $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

iv) If two vectors are parallel on antiparallel $\vec{A}$ x $\vec{B}$ = 0 $\hat{n}$ = a null vector As Sin 0° =0 & Sin 180° =0

v) CROSS Product of two equal vector is null vector. $\vec{A}$ X $\vec{A}$ = a null vector. As Sin 0° = 0 & Sin 180° = o

vi) $\vec{A}$ = Ax î + Ay ĵ + Az k̂ & $\vec{B}$ = Bx î + By ĵ + B_Z k̂ then $\vec{A}$ X $\vec{B}$

$$\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ Ax & Ay & Az \\ Bx & By & BZ \end{matrix}$$

or $\vec{A}$ X $\vec{B}$ = î (AyBz – AzBy) - ĵ (AxBz – AzBx) + k̂ (AxBy – AyBz)

vii) Magnitude of cross product of two vector is equal to area of Parallelogram formed with the two vectors as the two sides.

NEUMERICALS

1) What is the condition that A. B = $\vec{A}$ x $\vec{B}$

Let A.B = $\vec{A}$ X $\vec{B}$

=> AB Cos θ = AB Sin θ

=> Cos θ = Sin θ

It is possible only when 0 = 45° as Cos 45° = Sin 45°

2) If $\vec{A}$ = î - 2 ĵ - 5k̂ & $\vec{B}$ = 2 î + ĵ - 4k̂ find A X B

$$\vec{A} \text{ X } \vec{B} = \begin{matrix} i & j & k \\ 1 & -2 & -5 \\ 2 & 1 & -4 \end{matrix}$$

Or $\vec{A}$ X $\vec{B}$ = î (8 + 5) - ĵ (-4 +10) + k̂ (1 + 4)

$\vec{A}$ X $\vec{B}$ = 13 î - 6 ĵ + 5 k̂

4) A Particle gets displaced through 13 î - 6 ĵ + 5 k̂ due to a force 13 î - 6 ĵ + 5 k̂. The displacement and force are measure in MKS units. Find the Work done a force.

(Ans 20 joule)

5) Resolve a vector 5N which makes an angle 45° with X axis

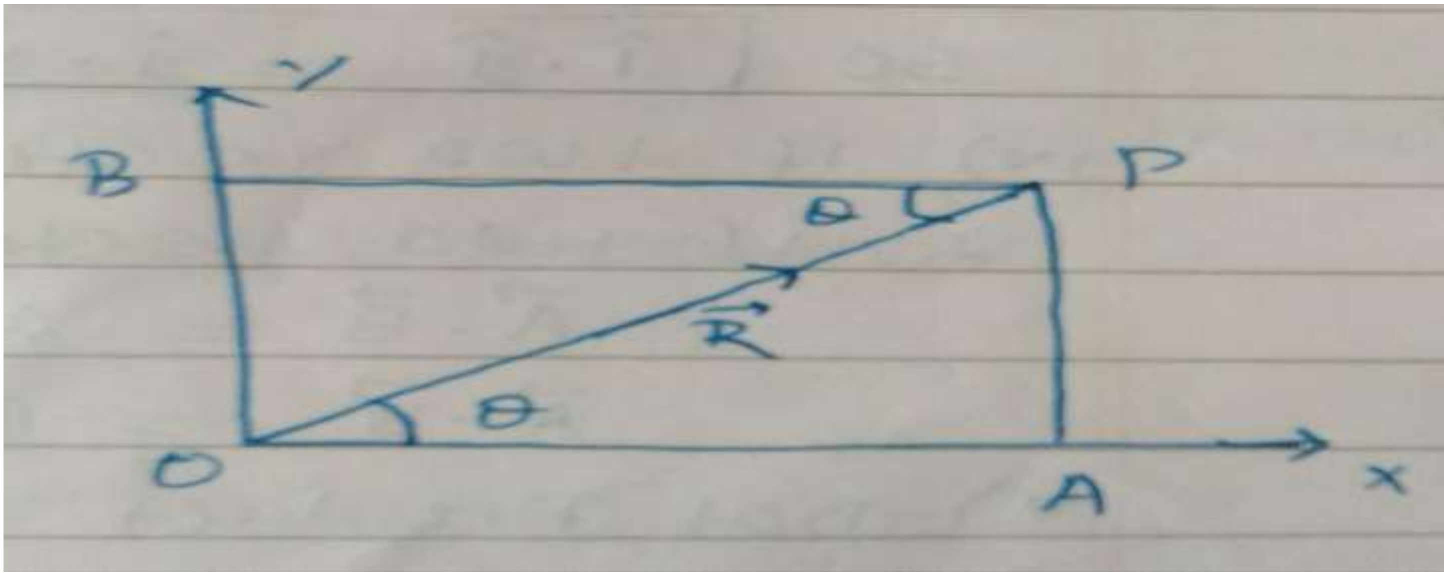
(Ans $\frac{5}{\sqrt{2}}$ & $\frac{5}{\sqrt{2}}$)

RESOLUTION OF VECTORS

Resolution of a vector is the process of obtaining the component of vector. Consider the $\vec{R}$= OP in XY plane. Draw a perpendicular from P to X-axis at A and another perpendicular from P to Y-axis at B. θ is the angle made by R with X-axis. In triangle OAP

COS θ = OA /OP => OA = R COS θ   equation 1

in triangle OBP



Sin θ = OB/OP => OB = R Sin θ   equation 2

Since OA = R_x & OB = R_y

R COS θ = Rx & R Sin θ = Ry

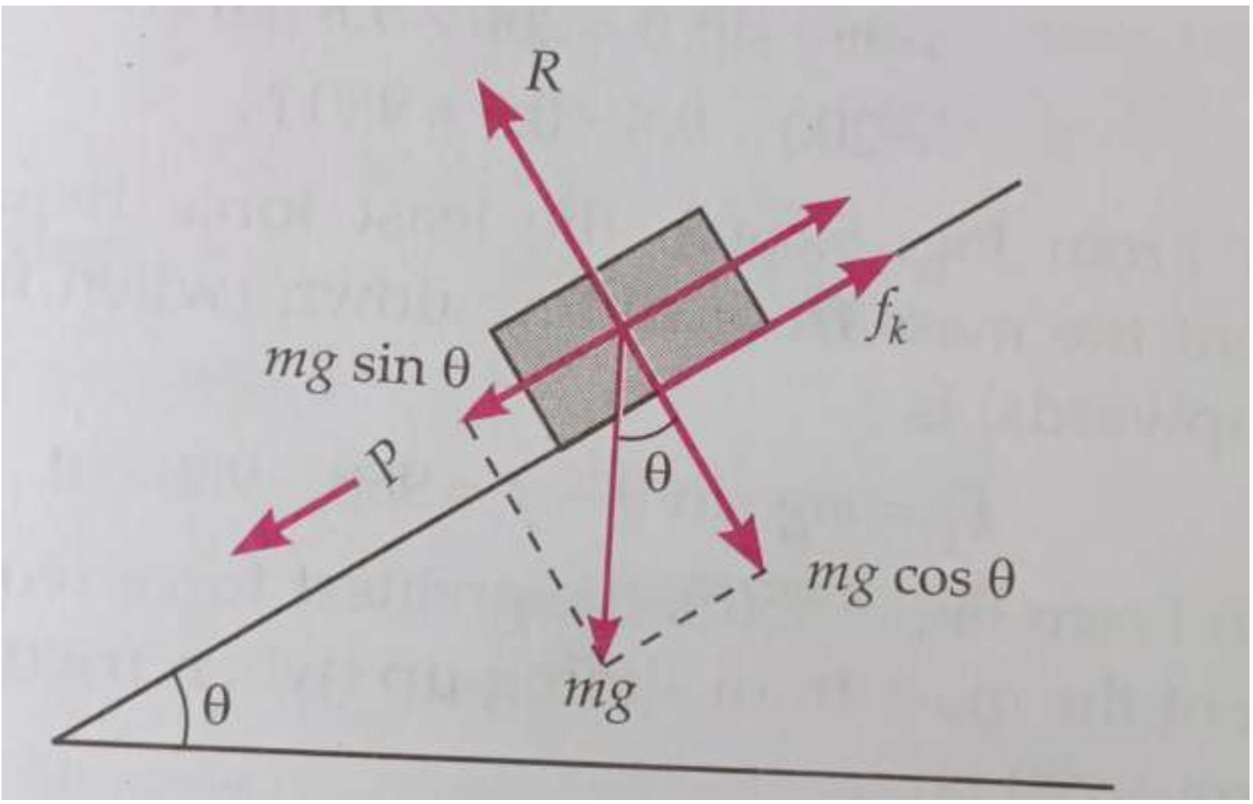
Or R =$\sqrt{(R_x^2+R_y^2)}$

APPLICATION TO INCLINED PLANE AND LAWN ROLLER

Suppose a body of weight mg is placed on an inclined plane, as shown in Fig. 5.82. with angle of inclination θThe weight mg of the body has two components:

(i) mg cosθ perpendicular to the inclined plane which balances the normal reaction R.

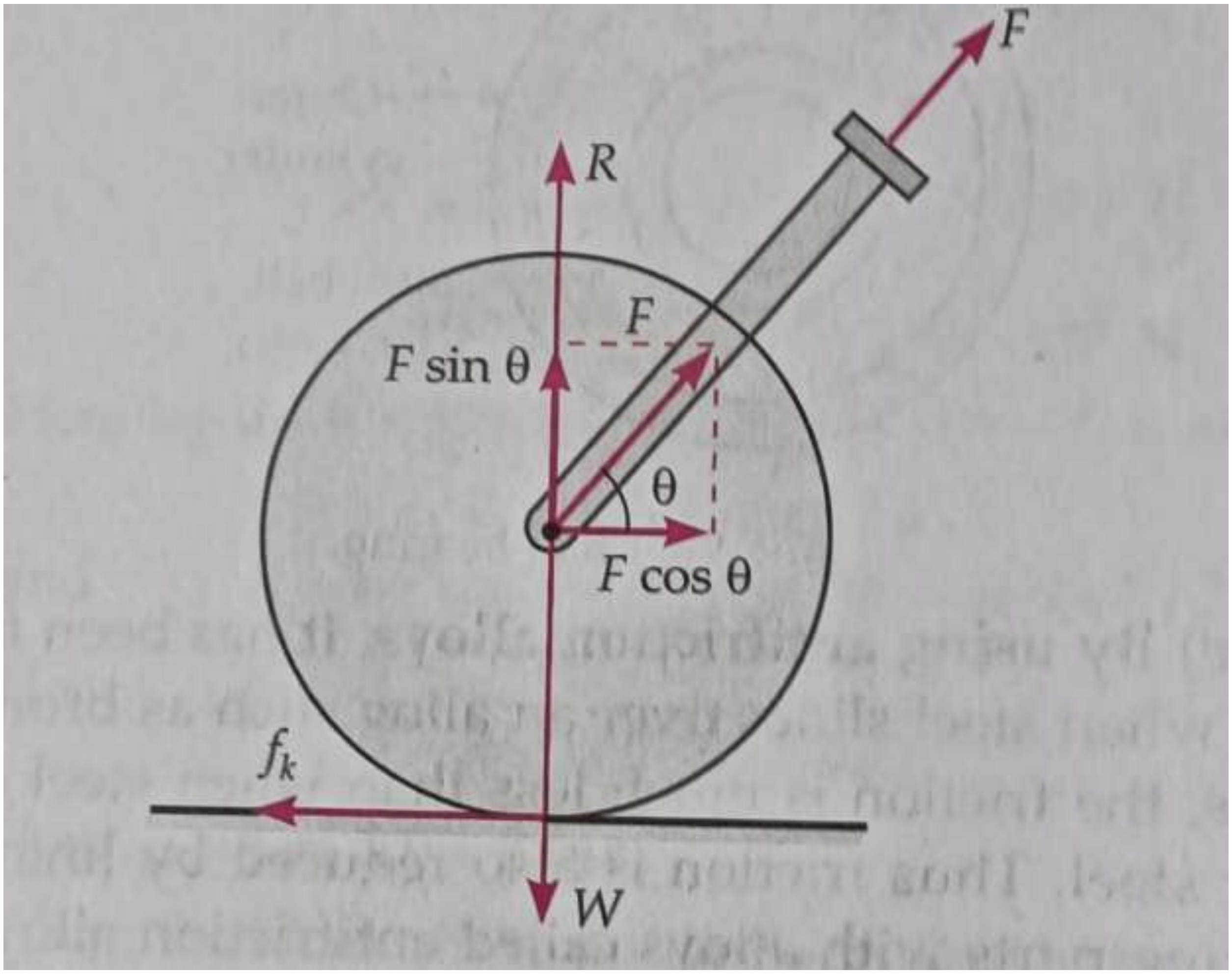
(ii) mg sinθ down the inclined plane.



Suppose a force F is applied to pull a lawn roller of weight W. The force F has two rectangular components:

(i) Horizontal component $F\cos\theta$ helps to move the roller forward.

(ii) Vertical component $F\sin\theta$ acts in the upward direction.



MOMENTUM

The amount of motion contained in a body is called momentum. It in denoted by P. It is a vector quantity.

Mathematically P=MV

where M = Mays of the body.

V = velocity of the body.

Its S I & C S units are gm cm/ sec^2 and kg met / sec^2

Its dimensional formula is $[M^1L^1T^{-1}$']

FORCE

The time rate of Change of momentum is called force. It is a vector quantity. It is denoted by $\vec{F}$

Mathematically $\vec{F} = \frac{dP}{dt}$

Or $\vec{F} = \frac{d}{dt}(M\vec{V})$

Or $\vec{F}$ = M $\frac{d}{dt}\vec{V}$ = M$\vec{a}$

In SI system 1f M=1 kg $\vec{a}$ = 1 m/sec² then F = 1 kg x 1 m/sec² = I newton

In CGS system if M=1 gm & a = 1cm/ sec^2 then F = 1gm XI cm / sec² = 1 dyne

1 newton = 10^5 dyne

Its dimensional formula is [M'L'T^{-2}]

LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for an isolated system (no external force acts on it), the total linear momentum of the system is conserved. The total linear momentum is the vector sum of the linear momenta of all the particles of the system.

Consider an isolated system of n particles.

Let $\vec{F}$ is the external force acting on the system, then according to Newton's second law $\vec{F}$= $\frac{d\vec{p}}{dt}$

For an isolated system, $\vec{F}$ = 0 or $\frac{d\vec{p}}{dt}$ = 0

It means that $\vec{p}$ = constant

Thus in the absence of any external force, the total linear momentum of the system is constant. This is the law of conservation of linear momentum.

RECOIL OF A GUN

Consider before firing, both the gun and the bullets are at rest.

.Let M → mass of the gun

m → mass of the bullet.

$\vec{V} \rightarrow velocity\ of\ gun\ after\ firing$

$\vec{v} \rightarrow velocity\ of\ bullet\ after\ firing$

As no external force acts on the system, so according to the principle of conservation of momentum, Total momentum before firing = Total momentum after firing

Or 0 = M$\vec{V}$ + m$\vec{v}$ or M$\vec{V}$ = - m$\vec{v}$

Or $\vec{V}$ = - $\frac{M}{m}\vec{v}$

The negative sign shows that $\vec{V}$ and $\vec{v}$ are in opposite directions.

Rocket and jetplanes work on the principle of conservation of momentum. Initially, both the rocket and its fuel are at rest. Their total momentum is zero. For rocket propulsion, the fuel is exploded. The burnt gases are allowed to escape through a nozzle with a very

W high downward velocity. The gases carry a large momentum in the downward direction. To conserve momentum, the rocket also acquires an equal momentum in which the upward direction and hence starts moving upwards.

UNIFORM CIRCULAR MOTION ANGULAR DISPLACEMENT, ANGULAR VELOCITY & ANGULAR ACCELERATION

If a particle is moving in such a way that its distance from a fixed Point is and always constant then the

Particle is said to be in circular motion. The path of the particle is called a circle. The fixed distance is the radius of the circle.

If a body moves in fixed circular orbit with same speed then it is said to be in uniform circular motion. If a body is in Uniform Circular motion then the angle through which the body get displaced is called ANGULAR DISPLACEMENT.

If $\Delta\theta$ = Angular displacement , $\Delta s =$ Linear displacement and r = radius of circle.

Then $\Delta\theta = \frac{\Delta s}{r}$

In vector form $\overrightarrow{\Delta\theta} = \frac{\overrightarrow{\Delta s}}{\vec{r}}$

The time rate of Change of angular displacement is called angular velocity. It is denoted by ω.

$\omega = \frac{\overrightarrow{\Delta\theta}}{\Delta t}$

Or $\omega = \frac{\Delta s}{\Delta t\, r} = \frac{\vec{v}}{r}$

Or $\vec{v} = \vec{\omega} \times \vec{r}$

Relation between Angular velocity and linear velocity.

The time rate of change of angular velocity is called ANGULAR ACCELERATION. It is denoted by α.

Mathematically α = dω /dt = (dv / dt) $\times$ (1 / r)

Or a = $\alpha\, r$

Relation between linear acceleration of angular acceleration.

where a = Linear acceleration.

In vector form $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \overrightarrow{v}$

CENTRIPETAL FORCE

A force required to make a body move along a circular path with uniform speed is called centripetal force. It always acts along the radius and towards the centre of the circular path.

When a body is in uniform circular motion, its velocity changes continuously due to change in the direction of motion. Hence it undergoes an acceleration which acts radially inwards. It is called centripetal acceleration and is given by a = $\frac{v^2}{r}$ = rω^2

where v and ω are the linear and angular speeds of the body and r is the radius of the circular path. According to Newton's second law

F = mass x centripetal acceleration

F= = m$\frac{v^2}{r}$ = mrω^2

Examples of centripetal force:

(i) For a stone rotated in a circle, the tension in the string provides the centripetal force.

(ii) The centripetal force for the motion of the planet around the sun is provided by the gravi ational force exerted by the sun on the planet.

(iii) For a car taking a circular turn on a horizontal road, the centripetal force is provided by the force of friction between the tyres and the road.

IMPULSE OF A FORCE

Impulse is the total effect of a large force which acts for a short time to produce a finite change in momentum. Impulse is defined as the product of the force and the time for which it acts and is equal to the total change in momentum.

Impulse = Force × time duration = Total change in momentum

Impulse is a vector quantity denoted by $\vec{J}$ Its direction is same as that of force or the change in momentum. The impulse of a force is positive, negative or zero depending on the momentum of the body increases, decreases or remains unchanged.

BANKING OF ROADS

To avoid skidding of vehicles at high speed turnings, roads are banked. As a result of this the normal reaction has a component which provides the requisite centripetal force . So the vehicle turns easily without overturning.

Let x → width of road

y → height of outer side over inner side

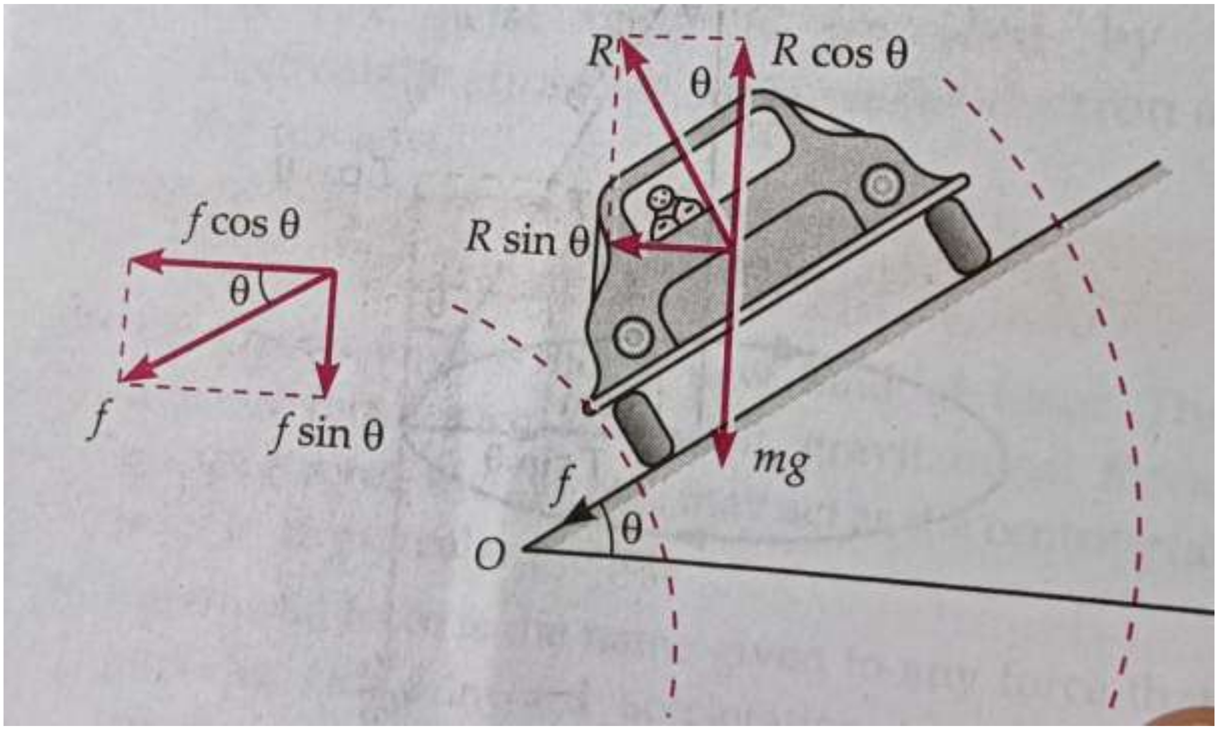
θ → angle of banking

W = mg → weight of vehicle

R → normal reaction

r → radius of circular track

f → force of friction



R is broken into R$\sin\theta$ which provides requisite centripetal force & R$\cos\theta$ which balances the weight of the vehicle.

Mathematically R$\cos\theta$ – f$\sin\theta$ = mg & R$\sin\theta$ + f$\cos\theta$ = $m\frac{v^2}{r}$

Or $\frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{v^2}{rg}$

$=> \frac{tan\theta + \frac{f}{R}}{1 - \frac{f}{R} tan\theta} = \frac{v^2}{rg}$

$=> \frac{tan\theta + \mu}{1 - \mu tan\theta} = \frac{v^2}{rg}$

If μ = 0 then tanθ = $\frac{v^2}{rg}$

Or $\theta = tan^{-1} \left(\frac{v^2}{rg} \right)$ Angle of banking

BENDING OF CYCLIST

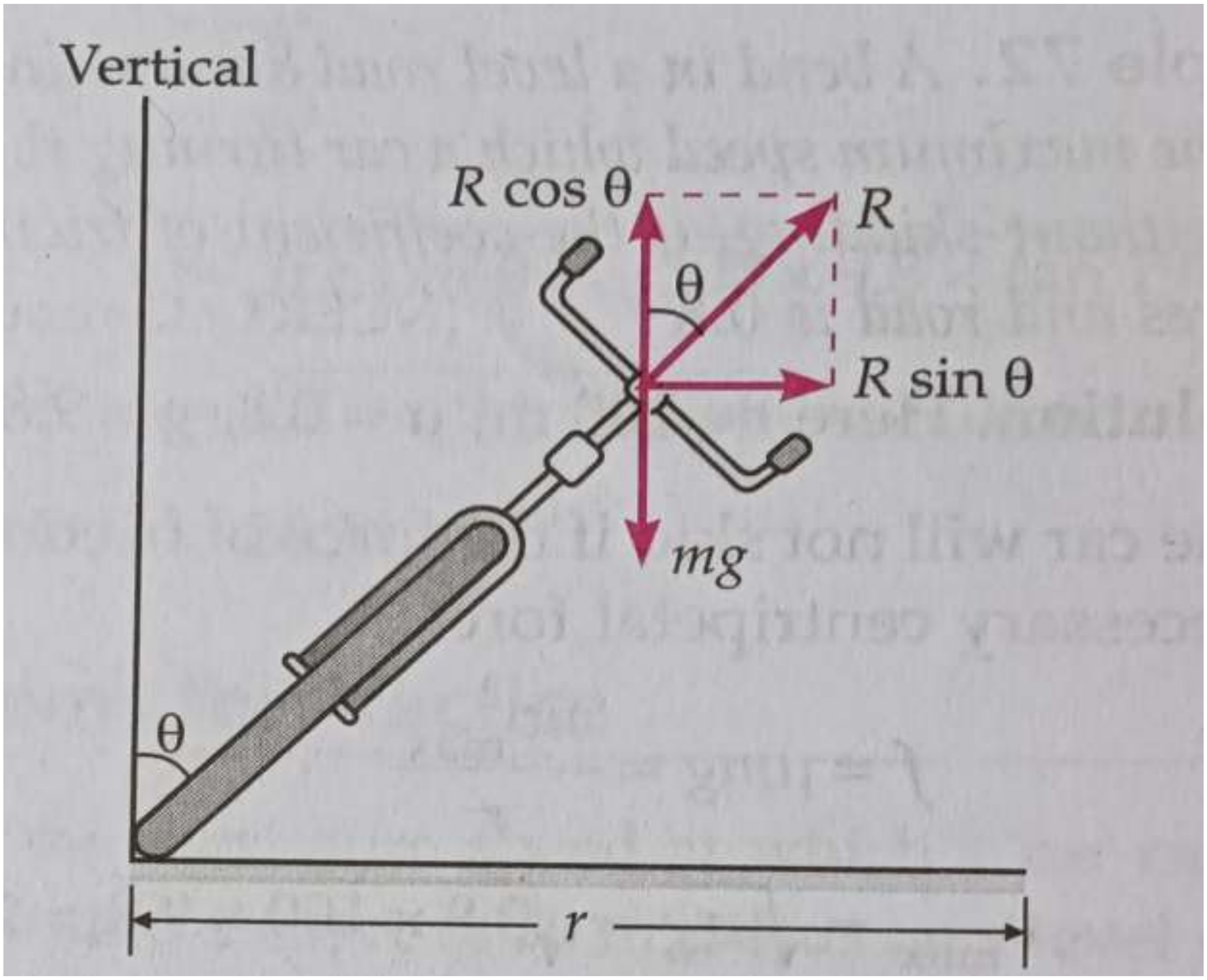
In case of a cyclist turning around a curved road, he bend a little bit from the vertical. So the reaction of the ground has a component which provides the necessary centripetal force. Suppose the cyclist bends inward through an angle of θ

Let $\theta \rightarrow$ angle of bending

W = mg $\rightarrow$ weight of vehicle

R $\rightarrow$ normal reaction

R is broken into R$\sin\theta$ which provides requisite centripetal force & R$\cos\theta$ which balances the weight of the cycle and cyclist.



Mathematically $R\cos\theta$ = mg & $R\sin\theta$ = $m\frac{v^2}{r}$

Or $\frac{\mathrm{R}\sin\theta}{\mathrm{R}\cos\theta} = \frac{v^2}{rg} => \tan\theta = \frac{v^2}{rg}$

Or $\theta = tan^{-1}\left(\frac{v^2}{rg}\right)$ Angle of bending.